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A note on the BPS spectrum of the matrix model*

Daniel Brace and Bogdan Morariu

Dept. of Physics, University of California, and
Theoretical Physics Group, Lawrence Berkeley National Laboratory,
University of California, Berkeley, California 94720
E-mail: brace@thwk2.lbl.gov, morariu@thsrv.lbl.gov

ABSTRACT: We calculate, using noncommutative supersymmetric Yang-Mills gauge theory, the part of the spectrum of the toroidally compactified Matrix theory which corresponds to quantized electric fluxes on two and three tori.

KEYWORDS: String Duality, M(atrix) Theories, Brane Dynamics in Gauge Theories.

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In this note we investigate the part of the spectrum corresponding to electric fluxes of the noncommutative supersymmetric Yang-Mills (NCSYM) gauge theory [1] compactified on a torus. This gives a description of the DLCQ of M-theory [2] compactified on a dual torus. Since the spectrum is invariant under the T-duality group $O(d,d\,|\mathbb{Z})$, where d is the dimension of the compactification torus, we can first calculate the spectrum in the simplest case which corresponds to a NCSYM gauge theory on a trivial bundle. Then we can use a duality transformation to rewrite the result in terms of the defining parameters of a dual theory on a nontrivial bundle. We will also obtain this result directly by quantizing the free system of collective coordinates of the twisted $U(n)$ theory.

To obtain the spectrum we mod out gauge equivalent configurations and show that the zero modes of the gauge field live on a torus. In the classical case one can find a global gauge transformation whose sole effect is a shift in the zero mode of the gauge field. Then the electric fluxes which are the conjugate variables are integrally quantized. However, for a nonvanishing deformation parameter the gauge transformation also results in a finite translation [3, 4, 5]. Then, just as for the electric charge of dyons [15], the electric flux spectrum, for states carrying momentum, contains an additional term proportional to the deformation parameter. We obtain a spectrum in agreement with similar calculations in the literature [3, 6, 7, 8, 9, 4, 5].

Throughout this paper we will use the same notation as in [10] where it was shown explicitly that the action of a $U(n)$ NCSYM, with magnetic fluxes M^{ij}, can be written as the action of a $U(q)$ NCSYM on a trivial quantum bundle, where q is the greatest common divisor of n and M^{ij}. Thus these two theories must have identical spectra. The noncommutative pure gauge theory action was written first in [1]. We can then introduce additional fields such that we obtain the maximally supersymmetric $U(n)$ NCSYM gauge theory [3, 11]

$$\begin{aligned}\mathcal{S}^{U(n)} = & \frac{1}{g^2_{SYM}}\int dt \int d^d\sigma \sqrt{\det(G^{kl})}\,\mathrm{tr}\Big(\frac{1}{2}G_{ij}\mathcal{F}^{0i}\mathcal{F}^{0j} - \\ & -\frac{1}{4}G_{ij}G_{kl}\left(\mathcal{F}^{ik}-\mathcal{F}^{ik}_{(0)}\right)\left(\mathcal{F}^{jl}-\mathcal{F}^{jl}_{(0)}\right)+\frac{1}{2}\sum_a \dot{X}^a\dot{X}^a - \\ & -\frac{1}{2}\sum_a G_{ij}\left[D^i,X^a\right]\left[D^j,X^a\right]+\frac{1}{4}\sum_{a,b}\left[X^a,X^b\right]\left[X^a,X^b\right]+\text{fermions}\Big)\,. \end{aligned} \quad (1)$$

As in [3, 11] the action contains magnetic backgrounds which we chose as in [10], so that the vacuum energy vanishes.

In general two NCSYM theories are dual to each other if there exists an element Λ of the duality group $SO(d,d\,|\mathbb{Z})$ with the block decomposition[1]

$$\Lambda = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}, \quad (2)$$

[1]The $SO(d,d\,|\mathbb{Z})$ subgroup of the T-duality group $O(d,d\,|\mathbb{Z})$ is the subgroup that does not exchange Type IIA and IIB string theories.

and a corresponding Weyl spinor representation matrix S, such that their defining parameters are related as follows

$$\begin{pmatrix} \bar{n} \\ \bar{M}^{23} \\ \bar{M}^{31} \\ \bar{M}^{12} \end{pmatrix} = S \begin{pmatrix} n \\ M^{23} \\ M^{31} \\ M^{12} \end{pmatrix}, \tag{3}$$

$$\bar{\Theta} = (\mathcal{A}\Theta + \mathcal{B})(\mathcal{C}\Theta + \mathcal{D})^{-1}, \tag{4}$$

$$\bar{g}^2_{SYM} = \sqrt{|\det(\mathcal{C}\Theta + \mathcal{D})|}\; g^2_{SYM}, \tag{5}$$

$$\bar{G}^{ij} = (\mathcal{C}\Theta + \mathcal{D})^i{}_k(\mathcal{C}\Theta + \mathcal{D})^j{}_l\, G^{kl}, \tag{6}$$

Equation (3) was written for the three dimensional case. Equation (4) was first written in [12]. A version of equation (3) appeared in [11] were S was identified as a canonical transformation. The transformation of the coupling constant (5) and the explicit transformation of the metric (6) were found in [10], where also the transformation (3) was identified as a chiral spinor transformation.

All the equations in this paper where d is unspecified, are valid for the two and three dimensional case, but some may have to be modified in higher dimensions. For simplicity we consider the case when n and M^{ij} are relatively prime. Then one can find a duality transformation Λ such that $\bar{n} = 1$ and $\bar{M} = 0$ as was shown in [10]. From this point on, when we discuss the $U(n)$ theory we will use the the d-dimensional block matrices (2), with Λ the particular transformation that takes the $U(n)$ theory into a $U(1)$ theory. For example the constant background field strength can be expressed in terms of the block components of Λ as

$$\mathcal{F}_{(0)} = \frac{-1}{2\pi}(\mathcal{C}\Theta + \mathcal{D})^{-1}\mathcal{C}\,. \tag{7}$$

We write the connection as a sum of a constant curvature $U(1)$ connection ∇_i, a zero mode $A^i_{(0)}$, and a fluctuating part A^i

$$\nabla^i - iA^i_{(0)}\mathbf{1} - iA^i(Z_j) = \partial^i + iF^{ij}\sigma_j - iA^i_{(0)}\mathbf{1} - iA^i(Z_j)\,.$$

Note that A^i does not contain the zero mode. The Z_i's are n-dimensional matrices which generate the algebra of adjoint sections. For example, in the two dimensional case we have [1, 3, 7, 13, 10]

$$Z_1 = e^{i\sigma_i Q^i_1/n}V^b\,, \qquad Z_2 = e^{i\sigma_i Q^i_2/n}U\,,$$

where U and V are the clock and shift matrices and Q is a two dimensional matrix which reduces to the identity in the commutative case. Substituting this in the action we obtain

$$\mathcal{S}^{U(n)} = \frac{1}{g^2_{SYM}}\int dt \int d^d\sigma\, \sqrt{\det(G^{kl})}\; \mathrm{tr}\, \frac{1}{2}G_{ij}\partial^0 A^i_{(0)}\partial^0 A^j_{(0)} + \cdots\,,$$

where the dots stand for terms containing only A^i. Thus classically the zero modes decouple, and the action is just that of a free particle

$$\mathcal{S}_{(0)}^{U(n)} = \int dt\, \frac{(2\pi)^2}{2}\, \mathcal{M}_{ij}\, \dot{A}^i_{(0)} \dot{A}^j_{(0)}\,,$$

where the mass matrix is given by

$$\mathcal{M}_{ij} = \left| n - \frac{1}{2}\mathrm{tr}(M\Theta)\right| \frac{(2\pi)^{d-2}\sqrt{\det(G^{kl})}}{g^2_{SYM}}\, G_{ij}\,. \tag{8}$$

In the commutative case the first factor on the right hand side of (8) reduces to n and arises from taking the trace. The origin of this factor in the noncommutative case was discussed in [3, 10]. The corresponding Hamiltonian is then[2]

$$\mathcal{H}_{(0)}^{U(n)} = \frac{1}{2}\mathcal{M}^{ij} E_i^{(0)} E_j^{(0)}\,, \tag{9}$$

where $\mathcal{M}^{ij}$ is the inverse mass matrix and $E_i^{(0)}$ is the momentum conjugate to $A^i_{(0)}$

$$E_i^{(0)} = \frac{1}{2\pi i}\,\frac{\partial}{\partial A_{(0)}}\,.$$

Note that $E_i^{(0)}$ correspond to zero modes of the electric field.

Before calculating the spectrum of (9) directly, we will use the duality invariance of the spectrum and obtain it by using the simpler dual $U(1)$ theory. We will use primes for all the variables in the $U(1)$ theory. In this case the mass matrix takes the form

$$\mathcal{M}'_{ij} = \frac{(2\pi)^{d-2}\sqrt{\det(G'^{kl})}}{g'^2_{SYM}}\, G'_{ij}\,. \tag{10}$$

Just as in the commutative $U(1)$ supersymmetric gauge theory [14] the zero modes live on a torus. To see this consider the gauge transformations[3]

$$U'_i = e^{i\sigma'_i}\,.$$

These gauge transformations are single valued and leave the trivial transition functions invariant. Under these gauge transformations the connection transforms as

$$U'^{-1}_j \left(\partial'^i - iA'^i_{(0)} - iA'^i(U'_k)\right) U'_j = \partial'^i - i\left(A'^i_{(0)} - \delta^i_j\right) - iA'^i\left(e^{-2\pi i\Theta'_{jk}} U'_k\right).$$

For vanishing Θ' the effect of these gauge transformations is just a shift of the zero mode and we have the following gauge equivalences $A'^i_{(0)} \sim A'^i_{(0)} + \delta^i_j$. Note

[2]This only includes the energy coming form the electric zero modes.

[3]We remind the reader that we use the notation of [10] were the definition of U_i differs from [3]

that $\delta^i_{(j)}$ for $j = 1, \ldots, d$ form a basis for a lattice L' and the configuration space is $\mathbf{R}^d/L'$. The conjugate momenta are then quantized

$$E_i'^{(0)} = n_i' \,,$$

and the spectrum of zero modes is then given by

$$\mathcal{E}^{U(1)} = \frac{1}{2}\mathcal{M}'^{ij} n_i' n_j' \,.$$

However in the noncommutative case we see that the above gauge transformations also produces a translation in the k direction proportional to Θ'_{jk}. This results in a modification of the spectrum similar to the Witten-Olive effect [15]. Let as define the total momentum operator operators P'_i such that

$$[P'^i, \Psi] = -i\,\partial'^i\Psi \,, \tag{11}$$

where Ψ is an arbitrary field of the theory. The momentum P'^i defined by (11) is not the standard gauge invariant total momentum but the difference between the two is the generator of a gauge transformation with the gauge parameter equal to the i-component of the gauge field. Thus on gauge invariant states the total momentum defined above and the gauge invariant momentum have the same effect.

The operator generating the gauge transformation is [4, 5]

$$\exp\left(2\pi i\left(E_j'^{(0)} + \Theta'_{jk}P'^k\right)\right). \tag{12}$$

Translation by an integral number of periods on a trivial bundle must leave the physical system invariant. The operators generating these translations are given by

$$\exp(2\pi i P'^k)\,. \tag{13}$$

The operators (12) and (13) act as the identity on physical states so we obtain the quantization

$$E_j'^{(0)} + \Theta'_{jk}P'^k = n_j' \,, \qquad P'^j = m'^j \,,$$

where n_j and m^j are integers. The spectrum of zero modes is then given by

$$\mathcal{E}^{U(1)} = \frac{1}{2}\mathcal{M}'^{ij}\left(n_i' - \Theta'_{ik}m'^k\right)\left(n_j' - \Theta'_{jl}m'^l\right).$$

We can describe this result in geometric terms. In the sectors of nonvanishing momentum the wave function for the zero modes is not strictly speaking a function but rather a section on a twisted bundle over the torus $\mathbf{R}^d/L'$ with twists given by $\exp\left(\Theta'_{ik}m'^k\right)$.

Using the duality transformations (6) we can express the spectrum in terms of the $U(n)$ parameters

$$\mathcal{E}^{U(n)} = \frac{1}{2}\,\frac{g^2_{SYM}}{(2\pi)^{d-2}\sqrt{\det(G^{ij})}}\Big|\det(\mathcal{C}\Theta + \mathcal{D})\Big|^{-1/2} G^{ij}\left(n_i - \Theta_{ik}m^k\right)\left(n_j - \Theta_{jk}m^k\right), \tag{14}$$

where we aslo performed a duality transformation on the quantum numbers [4, 5]

$$\begin{pmatrix} n'_i \\ m'^i \end{pmatrix} = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix} \begin{pmatrix} n_i \\ m^i \end{pmatrix} . \tag{15}$$

Next we consider in more detail the two dimensional case. The parameters of the $U(1)$ and $U(n)$ NCSYM are related by the $SO(2,2\,|\mathbb{Z})$ transformation [10]

$$\Lambda = \begin{pmatrix} aI_2 & b\varepsilon \\ -m\varepsilon & nI_2 \end{pmatrix} , \tag{16}$$

where ε is a two dimensional matrix with the only nonvanishing entries given by $\varepsilon_{12} = -\varepsilon_{21} = 1$. In this case $(\mathcal{C}\Theta + \mathcal{D})^i_{\ j} = (n + \theta m)\,\delta^i_j$ and the spectrum is

$$\mathcal{E}^{U(n)} = \frac{1}{2}\,\frac{g^2_{SYM}}{(2\pi)^{d-2}\,|n+\theta m|\sqrt{\det(G^{kl})}}\,G^{ij}(n_i + \theta m_i)(n_j + \theta m_j)\,,$$

where $m^i = \varepsilon^{ij} m_j$. This result[4] has the expected factor of $|n + \theta m|$ in the denominator. In the DLCQ formulation of M theory this factor is proportional to the kinetic momentum in the compact light-like direction and is expected to appear in the denominator of the DLCQ Hamiltonian.

Next we will obtain the spectrum directly in the $U(n)$ theory. We will do this in two ways. First, consider the generators of the adjoint algebra, the Z_i's. These generators satisfy

$$Z_k\left(\sigma_i + 2\pi\delta^j_i\right) = \Omega_j Z_k(\sigma_i)\Omega_j^{-1} . \tag{17}$$

Besides having the privileged role of generators for the sections of the adjoint bundle, the Z_i's can also be used to perform gauge transformations since they are unitary. We can rewrite (17) as

$$\Omega_j = Z_k\left(\sigma_i + 2\pi\delta^j_i\right)^{-1}\Omega_j Z_k(\sigma_i) . \tag{18}$$

The right hand side of (18) gives the transformation of the transition functions under the Z_i gauge transformation. We see that, just as in the $U(1)$ case where the gauge transformations U'_i left the transition functions trivial, the Z_i's leave the transition functions invariant. Following the same strategy as in the $U(1)$ case, where we used the U'_i to find the configuration space of the zero modes, we can use here Z_i

$$Z_j^{-1}\left(\nabla^i - iA^i_{(0)}\mathbf{1} - iA^i(Z_k)\right)Z_j =$$

$$\nabla^i - i\left(A^i_{(0)} - ((\mathcal{C}\Theta + \mathcal{D})^{-1})^i_{\ j}\right)\mathbf{1} - iA^i\left(e^{-2\pi i\Theta'_{jk}} Z_k\right) .$$

[4] Expressed in terms of the string coupling constant of the auxiliary string theory the spectrum takes the simpler form $E^{U(n)} = \frac{1}{2} g_s |n + \theta m|^{-1}\,G^{kl}(n_k + \theta m_k)(n_l + \theta m_l)$.

Note that again we have separated the zero mode of the gauge connection and we have used the identity [10]

$$[\nabla^i, Z_j] = i\left((\mathcal{C}\Theta+\mathcal{D})^{-1}\right)^i_{\ j} Z_j \,.$$

One can express the gauge transformed connection as

$$e^{-2\pi(\mathcal{A}\Theta+\mathcal{B})_{jk}\nabla^k}\left(\nabla^i - iA^i_{(0)}\mathbf{1} - iA^i(Z_k)\right)e^{2\pi(\mathcal{A}\Theta+\mathcal{B})_{jk}\nabla^k} + i\mathcal{A}_j^{\ i}\mathbf{1}\,, \tag{19}$$

where we used

$$(\mathcal{C}\Theta+\mathcal{D})^{-1} = (\mathcal{A}-\Theta'\mathcal{C})^T$$

and (7) to rewrite the extra shift in the zero mode.

Next we define the momentum operator by its action on the fields of the theory. For example on the gauge fields P^i acts as

$$\left[P^i, A^j_{(0)}\mathbf{1} + A^j(Z_k)\right] = -i\left[\nabla^i, A^j(Z_k)\right] - i\mathcal{F}^{ij}_{(0)}\,. \tag{20}$$

Note that P^i also acts on the zero mode $A^i_{(0)}$. This can be understood as follows. When we define the momentum we have the choice whether to include as part of the system the magnetic background $\mathcal{F}^{ij}_{(0)}$. The standard gauge invariant momentum for which the momentum density is $\mathrm{tr}(F^{ij}E_j)$ can be written as the sum of two terms. The first is just the momentum translating the part of the system that does not include the magnetic background and whose momentum density is $\mathrm{tr}((F^{ij}-F^{ij}_{(0)})E_j)$. The second term is an operator shifting the zero mode of the gauge field as in (20). Then our P^i can be identified, up to the generator of a gauge transformation, with the total momentum that includes the magnetic background. Furthermore, we can identify, up to the generator of a gauge transformation, the first term on the right hand side of (20) as the action of the momentum operator that translates only the fluctuating part. As we will see later it is the momentum whose density is $\mathrm{tr}(F^{ij}E_j)$ that appears in the $SO(d,d\,|\mathbb{Z})$ duality transformation.

A convenient way of writing the action of P^i on the gauge field is

$$\left[P^i, -iA^j_{(0)}\mathbf{1} - iA^j(Z_k)\right] = -i\left[\nabla^i, \nabla^j - iA^j_{(0)}\mathbf{1} - iA^j(Z_k)\right].$$

Then using (19) we see that the quantum operator which implements the gauge transformation above is given by

$$\exp\left(2\pi i(\mathcal{A}_j^{\ k}(E^{(0)}_k + \Theta_{kl}P^l) + \mathcal{B}_{jk}P^k)\right). \tag{21}$$

The momentum operator P^i has integer eigenvalues since the space is a torus with lengths 2π. One can also see this by considering the operator

$$\exp\left(2\pi i(\mathcal{C}^{ji}E^{(0)}_i + (\mathcal{C}^{jk}\Theta_{ki} + \mathcal{D}^j_{\ i})P^i)\right). \tag{22}$$

This acts trivially on every operator in the $U(n)$ theory. In particular the combination of operators in the exponent has no effect on the zero mode. The condition that (21) and (22) should act as the identity on the physical Hilbert space is equivalent to the quantization

$$\begin{aligned} \mathcal{A}_j^{\ k}\left(E_k^{(0)}+\Theta_{kl}P^l\right)+\mathcal{B}_{jk}P^k &= n'_j\,,\\ \mathcal{C}^{jk}\left(E_k^{(0)}+\Theta_{kl}P^l\right)+\mathcal{D}^j_{\ k}P^k &= m'^j\,. \end{aligned}$$

Since the matrices $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{D}$ are the block components of an element of $SO(d,d|\mathbb{Z})$ this is equivalent to

$$E_j^{(0)}+\Theta_{jk}P^k = n_j\,, \qquad P^j = m^j\,,$$

where n_j and m^j are integers. Using the Hamiltonian (9) and the above quantization the electric flux spectrum is

$$\begin{aligned} \mathcal{E}^{U(n)} &= \frac{1}{2}\,\frac{g^2_{SYM}}{(2\pi)^{d-2}\sqrt{\det(G^{ij})}}\left|n-\frac{1}{2}\mathrm{tr}(M\Theta)\right|^{-1}\times\\ &\quad\times G^{ij}\left(n_i-\Theta_{ik}m^k\right)\left(n_j-\Theta_{jk}m^k\right), \end{aligned} \qquad (23)$$

which is identical to the result (14) obtained by duality.

Finally we present an alternative derivation of the spectrum using the gauge transformations $\exp(i\bar{\sigma}_i)$ where $\bar{\sigma}_i = \sigma_j Q^j_{\ i}$ and $Q^j_{\ i}$ is a matrix defined in [10] and equals the identity for vanishing deformation parameter or magnetic background. This derivation is closely related to the derivation of the spectrum in [4, 5]. As discussed in [10] gauge invariant quantities such as the Lagrangian density have periodicity 2π in the $\bar{\sigma}_i$ variables. Then we can use $\bar{U}_i = \exp(i\bar{\sigma}_i)$ as a gauge transformation just as we used U'_i in the $U(1)$ theory. Note first that $\bar{U}_i$ is a globally defined gauge transformation. It is convenient to write it as $\bar{U}_i = \mathcal{U}_i\, e^{2\pi\Theta_{ij}\nabla^j}$. Here $\mathcal{U}_i = e^{i\sigma_i - 2\pi\Theta_{ij}\partial^j}$ and is the variable implementing the quotient condition [3]. The effect of this gauge transformation is

$$\begin{aligned} &\bar{U}_j^{-1}\left(\nabla^i - iA^i_{(0)}\mathbf{1} - iA^i(Z_k)\right)\bar{U}_j = \\ &\qquad = e^{-2\pi\Theta_{ij}\nabla^j}\left(\nabla^i - iA^i_{(0)}\mathbf{1} - iA^i(Z_k)\right)e^{2\pi\Theta_{ij}\nabla^j} + i\,\delta^i_{\ j}\,. \end{aligned} \qquad (24)$$

The operator implementing this gauge transformation in the Hilbert space is

$$\exp\left(-2\pi i\left(E_k^{(0)}+\Theta_{kl}P^l\right)\right).$$

Again, on gauge invariant states this operator acts trivially and together with the quantization of the momentum results in the same spectrum (23) as using Z_i. Note that the second method of deriving the $U(n)$ spectrum is similar in spirit to the derivation of the $U(1)$ spectrum. For example the gauge transformation is an element of the $U(1)$ subgroup. However, the first derivation is instructive since it exhibits inside the $U(n)$ theory the dual $U(1)$ theory variables such as P'^i and E'_i.

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References

[1] A. Connes and M. Rieffel, *Yang-Mills for noncommutative two-Tori*, *Operator algebras and mathematical physics*, Iowa City, Iowa 1985, 237; *Contemp. Math. Oper. Alg. Math. Phys.* **62**, AMS 1987.

[2] L. Susskind, *Another conjecture about M(atrix) theory*, `hep-th/9704080`.

[3] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [`hep-th/9711162`].

[4] C. Hofman, E. Verlinde, *U-duality of Born-Infeld on the noncommutative two-torus*, *J. High Energy Phys.* **12** (1998) 010 [`hep-th/9810116`].

[5] C. Hofman, E. Verlinde, *Gauge bundles and Born-Infeld on the noncommutative torus*, `hep-th/9810219`.

[6] N.A. Obers, B. Pioline and E. Rabinovici, *M-theory and U-duality on T^d with gauge backgrounds* *Nucl. Phys.* **B 525** (1998) 163 [`hep-th/9712084`].

[7] P.-M. Ho, *Twisted bundle on quantum torus and BPS states in matrix theory*, *Phys. Lett.* **B 434** (1998) 41 [`hep-th/9803166`].

[8] N.A. Obers and B. Pioline, *U-duality and M-theory*, `hep-th/9809039`.

[9] F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, *Noncommutative geometry from strings and branes*, `hep-th/9810072`.

[10] D. Brace, B. Morariu and B. Zumino, *Dualities of the matrix model from T-duality of the Type II string*, `hep-th/9810099`.

[11] A. Schwarz, *Morita equivalence and duality*, *Nucl. Phys.* **B 534** (1998) 720 [`hep-th/9805034`].

[12] M. Rieffel and A. Schwarz, *Morita equivalence of multidimensional noncommutative tori*, `math.QA/9803057`.

[13] B. Morariu and B. Zumino, *Super Yang-Mills on the noncommutative torus*, `hep-th/9807198`.

[14] E. Witten, *Constraints on supersymmetry breaking*, *Nucl. Phys.* **B 202** (1982) 253.

[15] E. Witten and D. Olive, *Supersymmetry algebras that include topological charges* *Phys. Lett.* **B 78** (1978) 97.